Weak and Strong forms of αĝ-irresolute functions

[1]R. Nagendran , [2]R. Latha and [3] S. Karpagam [4] C.Mayilan

[1,3]Department of Mathematics, Sree Sastha Institute of Engineering and Technology, Chennai-123. Tamil Nadu, India. E-mail: nagendranrajamani@gmail.com

[2]Department of Mathematics, Prince Shri Venkateshwara Padmavathy Engineering College, Chennai-100, Tamil Nadu, India.E-mail: ar.latha@gmail.com

[4]Department of Mathematics, Sree Sastha College of Engineering, Chennai-123. Tamil Nadu, India. E-mail: mayilan1975@gmail.com

Abstract

The purpose of this paper is to give two new types of irresolute functions called completely αĝ- irresolute functions and weakly αĝ-irresolute functions. We obtain their characterizations and basic properties.

2000 Mathematics Subject Classification: 54C10, 54C08, 54C05.
Keywords and phrases: αĝ-open set, αĝ-irresolute function, completely αĝ-irresolute function and weakly αĝ-irresolute function.

1.Introduction and Preliminaries

Functions and of course irresolute functions stand among the most important and most researched points in the whole of mathematical science. In 1972, Crossley and Hildebrand [3] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the years. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various areas of mathematics and related sciences.

Recently, as generalization of closed sets, the notion of αĝ-closed sets were introduced and studied by Abd El-Monsef et al [1]. In this paper, we will continue the study of related irresolute functions with αĝ-open sets. We introduce and characterize the concepts of completely αĝ-irresolute and weakly αĝ-irresolute functions.

Throughout this paper, spaces mean topological spaces on which no separation axioms are assumed unless otherwise mentioned and

f: $(X,\tau) \to (Y,\sigma)$ (or simply f: $X \to Y$) denotes a function f of a space (X,τ) into a space (Y,σ). Let A be a subset of a space X. The closure and the interior of A are denoted by cl(A) and int(A) respectively.

Definition 1.1 A subset A of a space X is called

(i) **regular open** [16] if A = int(clA);

(ii) **semi-open** [8] if $A \subseteq$ cl(intA);

(iii) **α-open** [12] if $A \subseteq$ int(cl(intA)).

The complement of regular open (resp. semi-open, α-open) is called regular closed (resp. semi-closed, α-closed).

The family of all regular open (resp. regular closed) subsets of X is denoted by RO(X) (resp. RC(X)). The α-closure of a subset A of X, denoted by αcl(A), is defined to be the intersection of all α-closed sets containing A.

Definition 1.2 A subset A of a space X is called:

(i) **ĝ-closed** [18] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

(ii) **αĝ-closed** [1] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ĝ-open in X.

The complement of ĝ-closed (resp. αĝ-closed) is called ĝ-open (resp. αĝ-open).

The family of all αĝ-open subsets of (X,τ) is denoted by $\alpha\hat{G}O(X)$. We set $\alpha\hat{G}O(X, x) = \{ V \in \alpha\hat{G}O(X) \setminus x \in V\}$ for $x \in X$ [14].

The union (resp. intersection) of all αĝ-open (resp. αĝ-closed) sets, each contained in (resp. containing) a set A in a space X is called the αĝ-interior (resp. αĝ-closure) of A and is denoted by αĝ-int(A) (resp. αĝ-cl(A))[14].

Definition 1.3 A function f: $X \to Y$ is called:

(i) strongly continuous [9] if $f^{-1}(V)$ is both open and closed in X for each subset V of Y;

(ii) completely continuous [2] if $f^{-1}(V)$ is regular open in X for each open subset V of Y;

(iii) αĝ-irresolute [14] if $f^{-1}(V)$ is αĝ-closed in X for each αĝ-closed subset V of Y and

(iv) pre αĝ-closed [14] if f(V) is αĝ-closed in Y for each αĝ-closed subset V of X.

2. Completely $\alpha\hat{g}$-irresolute functions

Definition 2.1 A function f: X → Y is called completely $\alpha\hat{g}$-irresolute if the inverse image of each $\alpha\hat{g}$-open subset of Y is regular open in X.

Clearly, every strongly continuous function is completely $\alpha\hat{g}$-irresolute and every completely $\alpha\hat{g}$-irresolute function is $\alpha\hat{g}$-irresolute.

Remark 2.2 The converses of the above implications are not true in general as seen from the following examples.

Example 2.3 Let X = Y = {a, b, c}, $\tau = \sigma = \{\phi, X = Y, \{a\}, \{b,c\}\}$. Then the identity function f: $(X,\tau) \to (Y,\sigma)$ is completely $\alpha\hat{g}$-irresolute but not strongly continuous.

Example 2.4 Let X = Y = { a, b, c}, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\phi, Y, \{a, b\}\}$. Then the identity function f: $(X,\tau) \to (Y,\sigma)$ is $\alpha\hat{g}$-irresolute but not completely $\alpha\hat{g}$-irresolute.

Theorem 2.5 The following statements are equivalent for a function f: X → Y.

(i) f is completely $\alpha\hat{g}$-irresolute.

(ii) f: $(X,\tau) \to (Y, \alpha\hat{G}O(X))$ is completely continuous.

(iii) $f^{-1}(F)$ is regular closed in X for each $\alpha\hat{g}$-closed set F of Y.

Proof

(i) ⇔ (ii) : It follows from the definitions.

(i) ⇒ (iii) : Let F be any $\alpha\hat{g}$-closed set of Y. Then $Y\backslash F \in \alpha\hat{G}O(Y)$. By (i), $f^{-1}(Y\backslash F) = X\backslash f^{-1}(F) \in RO(X)$. We have $f^{-1}(F) \in RC(X)$.

The converse is similar.

Lemma 2.6 [10] Let S be an open subset of a space (X,τ). Then the following hold.

(i) If U is regular open in X, then so is U ∩ S in the subspace (S,τ_S).

(ii) If $B \subset S$ is regular open in (S,τ_S), then there exists a regular open set U in (X,τ) such that B = U ∩ S.

Theorem 2.7 If f: $(X,\tau) \to (Y,\sigma)$ is a completely $\alpha\hat{g}$-irresolute function and A is any open subset of X, then the restriction $f|A$: A → Y is completely $\alpha\hat{g}$-irresolute.

Proof Let F be a $\alpha\hat{g}$-open subset of Y. By hypothesis, $f^{-1}(F)$ is regular open in X. Since A is open in X. It follows from the previous lemma that

$(f|A)^{-1}(F) = A \cap f^{-1}(F)$ which is regular open in A. Therefore, $f|A$ is completely $\alpha\hat{g}$-irresolute.

Definition 2.8 A function f: X → Y is called strongly $\alpha\hat{g}$-continuous if $f^{-1}(V)$ is $\alpha\hat{g}$-open in X for each subset V of Y.

Theorem 2.9 The following hold for the functions f: $(X,\tau) \rightarrow (Y,\sigma)$ and g: $(Y,\sigma) \rightarrow (Z,\rho)$:

(i) If f is completely $\alpha\hat{g}$-irresolute and g is strongly $\alpha\hat{g}$-continuous, then g o f : $(X,\tau) \rightarrow (Z,\rho)$ is completely continuous.

(ii) If f is completely $\alpha\hat{g}$-irresolute and g is $\alpha\hat{g}$-irresolute, then g o f : $(X,\tau) \rightarrow (Z,\rho)$ is completely $\alpha\hat{g}$-irresolute.

(iii) If f is completely continuous and g is completely $\alpha\hat{g}$-irresolute, then g o f : $(X,\tau) \rightarrow (Z,\rho)$ is completely $\alpha\hat{g}$-irresolute.

Proof The proof of the theorem is easy and hence omitted.

Definition 2.10 A space X is said to be almost connected [5] (resp. $\alpha\hat{g}$-connected [14]) if there does not exist disjoint regular open (resp. $\alpha\hat{g}$-open) sets A and B such that A U B = X.

Theorem 2.11 If f: X → Y is completely $\alpha\hat{g}$-irresolute surjective function and X is almost connected, then Y is $\alpha\hat{g}$-connected.

Proof Suppose that Y is not $\alpha\hat{g}$-connected. Then there exist disjoint $\alpha\hat{g}$-open sets A and B of Y such that A U B = X. Since f is completely $\alpha\hat{g}$-irresolute surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are regular open sets in X. Moreover, $f^{-1}(A) \cup f^{-1}(B) = X$, $f^{-1}(A) \neq \phi$ and $f^{-1}(B) \neq \phi$. This shows that X is not almost connected, which is a contradiction to the assumption that X is almost connected. By contradiction, Y is $\alpha\hat{g}$-connected.

Definition 2.12 A space X is said to be

(i) **nearly compact** [13, 15] if every regular open cover of X has a finite subcover;

(ii) **nearly countably compact** [7] if every countable cover by regular open sets has a finite subcover;

(iii) **nearly Lindelőf** [6] if every cover of X by regular open sets has a countable subcover;

(iv) **$\alpha\hat{g}$-compact** [14] if every $\alpha\hat{g}$-open cover of X has a finite subcover;

(v) **countably $\alpha\hat{g}$-compact** [14] if every $\alpha\hat{g}$-open countable cover of X has a finite subcover and

(vi) **$\alpha\hat{g}$-Lindelőf** [14] if every cover of X by $\alpha\hat{g}$-open sets has a countable subcover.

Theorem 2.13 Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a completely $\alpha\hat{g}$-irresolute surjective function. Then the following statements hold:

(i) If X is nearly compact, then Y is $\alpha\hat{g}$-compact.

(ii) If X is nearly Lindelőf, then Y is $\alpha\hat{g}$-Lindelőf.

(iii) If X is nearly countably compact, then Y is countably $\alpha\hat{g}$-compact.

Proof (i) Let f: $X \rightarrow Y$ be a completely $\alpha\hat{g}$-irresolute function of nearly compact space X onto a space Y. Let $\{U_\alpha : \alpha \in \Delta\}$ be any $\alpha\hat{g}$-open cover of Y. Then $\{ f^{-1}(U_\alpha) : \alpha \in \Delta\}$ is a regular open cover of X. Since X is nearly compact, there exists a finite subfamily $\{ f^{-1}(U_{\alpha i}) / i =1,2,\ldots.n\}$ of $\{ f^{-1}(U_\alpha) : \alpha \in \Delta\}$ which cover X. It follows that $\{U_{\alpha i} : i =1,2,\ldots.n\}$ is a finite subfamily of $\{U_\alpha : \alpha \in \Delta\}$ which cover Y. Hence, space Y is a $\alpha\hat{g}$-compact space.

The proof of other cases are similar.

Definition 2.14 A space (X,τ) is said to be

(i) **S-closed** [17] (resp. $\alpha\hat{g}$-closed compact [14]) if every regular closed (resp. $\alpha\hat{g}$-closed) cover of X has a finite subcover;

(ii) **Countably S-closed compact** [4] (resp. countably $\alpha\hat{g}$-closed compact [14]) if every countable cover of X by regular closed (resp. $\alpha\hat{g}$-closed) sets has a finite subcover;

(iii) **S-Lindelőf** [11] (resp. $\alpha\hat{g}$-closed Lindelőf [14]) if every cover of X by regular closed (resp.$\alpha\hat{g}$-closed) sets has a countable subcover.

Theorem 2.15 Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a completely $\alpha\hat{g}$-irresolute surjective function. Then the following statements hold:

(i) If X is S-closed, then Y is $\alpha\hat{g}$-closed compact.

(ii) If X is S-Lindelőf, then Y is $\alpha\hat{g}$-closed Lindelőf.

(iii) If X is countably S-closed compact, then Y is countably $\alpha\hat{g}$-closed compact.

Proof It can be obtained similarly as the previous theorem.

Definition 2.16 A space X is said to be strongly $\alpha\hat{g}$-normal (resp. mildly $\alpha\hat{g}$-normal) if for each pair of distinct $\alpha\hat{g}$-closed (resp. regular closed) sets A and B of X, there exist disjoint $\alpha\hat{g}$-open sets U and V such that $A \subset U$ and
$B \subset V$.

It is evident that every strongly $\alpha\hat{g}$-normal space is mildly $\alpha\hat{g}$-normal.

In [14], the following theorem is proved.

Theorem 2.17 If a map f: X → Y is pre αĝ-closed, then for each subset B of Y and each αĝ-open set U of X containing $f^{-1}(B)$, there exists a αĝ-open set V in Y containing B such that $f^{-1}(V) \subset U$.

Theorem 2.18 If f: X → Y is completely αĝ-irresolute, pre αĝ-closed function from a mildly αĝ-normal space X onto a space Y, then Y is strongly αĝ-normal.

Proof Let A and B be two disjoint αĝ-closed subsets of Y. Then, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint regular closed subsets of X. Since X is mildly αĝ-normal space, there exist disjoint αĝ-open sets U and V in X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Then by Theorem 2.17, there exist αĝ-open sets G = Y \ f (X\U) and H = Y \ f (X\V) such that $A \subset G$, $f^{-1}(G) \subset U$; $B \subset H$, $f^{-1}(H) \subset V$. Clearly, G and H are disjoint αĝ-open subsets of Y. Hence Y is strongly αĝ-normal.

Now, we define the following.

Definition 2.19 A space X is said to be strongly αĝ-regular if for each αĝ-closed set F and each point $x \notin F$, there exist disjoint αĝ-open sets U and V in X such that $x \in U$ and $F \subset V$.

Definition 2.20 A space X is called almost αĝ-regular if for each regular closed subset F and every point $x \notin F$, there exist disjoint αĝ-open sets U and V such that $x \in U$ and $F \subset V$.

Theorem 2.21 If f is a completely αĝ-irresolute, pre αĝ-closed injection of an almost αĝ-regular space X onto a space Y, then Y is strongly αĝ-regular space.

Proof Let F be a αĝ-closed subset of Y and let $y \notin F$. Then, $f^{-1}(F)$ is regular closed subset of X such that $f^{-1}(y) = x \notin f^{-1}(F)$. Since X is almost αĝ-regular space, there exist disjoint αĝ-open sets U and V in X such that$f^{-1}(y) \in U$ and $f^{-1}(F) \subset V$. By Theorem 2.17, there exist αĝ-open setsG = Y \ f (X\U) such that $f^{-1}(G) \subset U$, $y \in G$ and H = Y \ f(X\V) such that $f^{-1}(H) \subset V$, $F \subset H$. Clearly, G and H are disjoint αĝ-open subsets of Y. Hence, Y is strongly αĝ-regular space.

Definition 2.22 A space (X,τ) is said to be αĝ-T_1 [14] (resp. r-T_1 [6]) if for each pair of distinct points x and y of X, there exist αĝ-open (resp.regular open) sets U1 and U2 such that $x \in U_1$ and $y \in U2$, $x \notin U2$ and$y \notin U1$.

Theorem 2.23 If f: (X,τ) → (Y,σ) is completely αĝ-irresolute injective function and Y is αĝ-T_1, then X is r-T_1.

Proof Suppose that Y is αĝ-T_1. For any two distinct points x and y of X, there exist αĝ-open sets F_1 and F2 in Y such that $f(x) \in F_1$, $f(y) \in F2$, $f(x) \notin F2$ and $f(y) \notin F_1$. Since f is injective completely αĝ-irresolute function, we have X is r-T_1.

Definition 2.24 A space (X,τ) is said to be αĝ-T_2 [14] if for each pair of distinct points x and y in X, there exist disjoint αĝ-open sets A and B in X such that $x \in A$ and $y \in B$ and $A \cap B = \phi$.

Theorem 2.25 If f: $(X,\tau) \to (Y,\sigma)$ is completely $\alpha\hat{g}$-irresolute injective function and Y is $\alpha\hat{g}$-T_2, then X is T_2.

Proof Similar to the proof of Theorem 2.23.

3. Weakly $\alpha\hat{g}$-irresolute functions

Definition 3.1 A function f: X → Y is said to be weakly $\alpha\hat{g}$-irresolute if for each point $x \in X$ and each $V \in \alpha\hat{G}O(Y, f(x))$, there exists a $U \in \alpha\hat{G}O(X, x)$ such that $f(U) \subset \alpha\hat{g}\text{-cl}(V)$.

It is evident that every $\alpha\hat{g}$-irresolute function is weakly $\alpha\hat{g}$-irresolute but the converse is not true.

Example 3.2 Let X = Y = { a, b, c}, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$. Define a function f: $(X,\tau) \to (Y,\sigma)$ by f(a) = c, f(b) = b, f(c) = a. Clearly f is weakly $\alpha\hat{g}$-irresolute but not $\alpha\hat{g}$-irresolute.

Theorem 3.3 For a function f: X → Y, the following statements are equivalent:

(i) f is weakly $\alpha\hat{g}$-irresolute.

(ii) $f^{-1}(V) \subset \alpha\hat{g}\text{-int}(f^{-1}(\alpha\hat{g}\text{-cl}(V)))$ for every $V \in \alpha\hat{G}O(Y)$.

(iii) $\alpha\hat{g}\text{-cl}(f^{-1}(V)) \subset f^{-1}(\alpha\hat{g}\text{-cl}(V))$ for every $V \in \alpha\hat{G}O(Y)$.

Proof

(i) ⇒ (ii) : Suppose that $V \in \alpha\hat{G}O(Y)$ and let $x \in f^{-1}(V)$. It follows from (i) that $f(U) \subset \alpha\hat{g}\text{-cl}(V)$ for some $U \in \alpha\hat{G}O(X, x)$. Therefore, we have $U \subset f^{-1}(\alpha\hat{g}\text{-cl}(V))$ and $x \in U \subset \alpha\hat{g}\text{-int}(f^{-1}(\alpha\hat{g}\text{-cl}(V)))$. This shows that $f^{-1}(V) \subset \alpha\hat{g}\text{-int}(f^{-1}(\alpha\hat{g}\text{-cl}(V)))$.

(ii) ⇒ (iii) : Suppose that $V \in \alpha\hat{G}O(Y)$ and $x \notin f^{-1}(\alpha\hat{g}\text{-cl}(V))$. Then $f(x) \notin \alpha\hat{g}\text{-cl}(V)$. There exists $G \in \alpha\hat{G}O(Y, f(x))$ such that $G \cap V = \phi$. Since $V \in \alpha\hat{G}O(Y)$, we have $\alpha\hat{g}\text{-cl}(G) \cap V = \phi$ and hence $\alpha\hat{g}\text{-int}(f^{-1}(\alpha\hat{g}\text{-cl}(G))) \cap f^{-1}(V) = \phi$. By (ii), we have $x \in f^{-1}(G) \subset \alpha\hat{g}\text{-int}(f^{-1}(\alpha\hat{g}\text{-cl}(G))) \in \alpha\hat{G}O(X)$. Therefore, we obtain $x \notin \alpha\hat{g}\text{-cl}(f^{-1}(V))$. This shows that $\alpha\hat{g}\text{-cl}(f^{-1}(V)) \subset f^{-1}(\alpha\hat{g}\text{-cl}(V))$.

(iii) ⇒ (i): Let $x \in X$ and $V \in \alpha\hat{G}O(Y, f(x))$. Then $x \notin f^{-1}(\alpha\hat{g}\text{-cl}(Y \setminus \alpha\hat{g}\text{-cl}(V)))$. Since $Y \setminus \alpha\hat{g}\text{-cl}(V) \in \alpha\hat{G}O(Y)$, by (iii), we have $x \notin \alpha\hat{g}\text{-cl}(f^{-1}(Y \setminus \alpha\hat{g}\text{-cl}(V)))$. Hence there exists $U \in \alpha\hat{G}O(X, x)$ such that $U \cap f^{-1}(Y \setminus \alpha\hat{g}\text{-cl}(V)) = \phi$. Therefore, we obtain $f(U) \cap (Y \setminus \alpha\hat{g}\text{-cl}(V)) = \phi$ and hence $f(U) \subset \alpha\hat{g}\text{-cl}(V)$. This shows that f is weakly $\alpha\hat{g}$-irresolute.

Theorem 3.4 A function f: X → Y is weakly $\alpha\hat{g}$-irresolute if the graph function, defined by g(x) = (x, f(x)) for each $x \in X$, is weakly $\alpha\hat{g}$-irresolute.

Proof Let $x \in X$ and $V \in \alpha\hat{G}O(Y, f(x))$. Then X x V is a $\alpha\hat{g}$-open set of X x Y containing g(x). Since g is weakly $\alpha\hat{g}$-irresolute, there exists $U \in \alpha\hat{G}O(X, x)$ such that $g(U) \subset \alpha\hat{g}\text{-cl}(X \times V) \subset X \times \alpha\hat{g}\text{-cl}(V)$. Therefore,we have $f(U) \subset \alpha\hat{g}\text{-cl}(V)$.

Theorem 3.5 [14] A space X is $\alpha\hat{g}$-T_2 if and only if for any pair of distinct points x, y of X there exist $\alpha\hat{g}$-open sets U and V such that $x \in U$ and $y \in V$ and $\alpha\hat{g}\text{-cl}(U) \cap \alpha\hat{g}\text{-cl}(V) = \phi$.

Theorem 3.6 If Y is a $\alpha\hat{g}$-T_2 space and f: X → Y is a weakly $\alpha\hat{g}$-irresolute injection, then X is $\alpha\hat{g}$-T_2.

Proof Let x, y be any two distinct points of X. Since f is injective , we have $f(x) \neq f(y)$. Since Y is $\alpha\hat{g}$-T_2, by Theorem 3.5 there exist $V \in \alpha\hat{G}O(Y, f(x))$ and $W \in \alpha\hat{G}O(Y, f(y))$ such that $\alpha\hat{g}\text{-cl}(V) \cap \alpha\hat{g}\text{-cl}(W) = \phi$. Since f is weakly $\alpha\hat{g}$-irresolute, there exist $G \in \alpha\hat{G}O(X, x)$ and $H \in \alpha\hat{G}O(X, y)$ such that $f(G) \subset \alpha\hat{g}\text{-cl}(V)$ and $f(H) \subset \alpha\hat{g}\text{-cl}(W)$. Hence we obtain $G \cap H = \phi$. This shows that X is $\alpha\hat{g}$-T_2.

Definition 3.7 A function f: X → Y is said to have a strongly $\alpha\hat{g}$-closed graph if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in \alpha\hat{G}O(X, x)$ and $V \in \alpha\hat{G}O(Y, y)$ such that $(\alpha\hat{g}\text{-cl}(U) \times \alpha\hat{g}\text{-cl}(V)) \cap G(f) = \phi$.

Theorem 3.8 If Y is a $\alpha\hat{g}$-T_2 space and f: X → Y is weakly $\alpha\hat{g}$-irresolute, then G(f) is strongly $\alpha\hat{g}$-closed.

Proof Let $(x, y) \in (X \times Y)\backslash G(f)$. Then $y \neq f(x)$ and by Theorem 3.5 there exist $V \in \alpha\hat{G}O(Y, f(x))$ and $W \in \alpha\hat{G}O(Y, y)$ such that $\alpha\hat{g}\text{-cl}(V) \cap \alpha\hat{g}\text{-cl}(W) = \phi$. Since f is weakly $\alpha\hat{g}$-irresolute, there exists $U \in \alpha\hat{G}O(X, x)$ such that $f(\alpha\hat{g}\text{-cl}(U)) \subset \alpha\hat{g}\text{-cl}(V)$. Therefore, we obtain $f(\alpha\hat{g}\text{-cl}(U)) \cap \alpha\hat{g}\text{-cl}(W) = \phi$ and hence $(\alpha\hat{g}\text{-cl}(U) \times \alpha\hat{g}\text{-cl}(W)) \cap G(f) = \phi$. This shows that G(f) is strongly $\alpha\hat{g}$-closed in X x Y.

Theorem 3.9 If a function f: X → Y is weakly $\alpha\hat{g}$-irresolute, injective and G(f) is strongly $\alpha\hat{g}$-closed, then X is $\alpha\hat{g}$-T_2.

Proof Let x and y be a pair of distinct points of X. Since f is injective, $f(x) \neq f(y)$ and $(x, f(y)) \notin G(f)$. Since G(f) is strongly $\alpha\hat{g}$-closed, there exist$G \in \alpha\hat{G}O(X, x)$ and $V \in \alpha\hat{G}O(Y, f(y))$ such that $f(\alpha\hat{g}\text{-cl}(G)) \cap \alpha\hat{g}\text{-cl}(V) = \phi$.Since f is weakly $\alpha\hat{g}$-irresolute, there exists $H \in \alpha\hat{G}O(X, y)$ such that $f(H) \subset \alpha\hat{g}\text{-cl}(V)$. Hence we have $f(\alpha\hat{g}\text{-cl}(G)) \cap f(H) = \phi$; hence $G \cap H = \phi$. This shows that X is $\alpha\hat{g}$-T_2.

Theorem 3.10 If a function f: X → Y is a weakly $\alpha\hat{g}$-irresolute surjection and X is $\alpha\hat{g}$-connected, then Y is $\alpha\hat{g}$-connected.

Proof Suppose that Y is not $\alpha\hat{g}$-connected. There exist non-empty $\alpha\hat{g}$-open sets V and W of Y such that $V \cup W = Y$ and $V \cap W = \phi$. Since f is weakly $\alpha\hat{g}$-irresolute, $f^{-1}(V), f^{-1}(W) \in \alpha\hat{G}O(X)$. Moreover, we have $f^{-1}(V) \cup f^{-1}(W) = X$, $f^{-1}(V) \cap f^{-1}(W) = \phi$ and $f^{-1}(V)$ and $f^{-1}(W)$ are non-empty. Therefore, X is not $\alpha\hat{g}$-connected.

References

1. M.E. Abd El-Monsef, S. Rosemary and M.L. Thivagar, **$\alpha\hat{G}$-closed sets** in Topological Spaces, Assiut Univ. J. of Mathematics and Computer Science, 36(1) (2007), 43-51.
2. S.P. Arya and R. Gupta, On strongly continuous maps, Kyungpook Math. J. 14 (1974), 131-143.
3. S.G. Crossley and S.K. Hildebrand, Semi-Topological properties, Fund. Math. 74 (1972), 233-254.
4. K. Dlaska, N. Ergun and M. Ganster, Countable S-closed spaces, Math. Slovaca, 44 (1994), 337-348.
5. E. Ekici, Generalization of perfectly continuous, Regular set- connected and clopen functions, Acta. Math. Hungar., 107 (3) (2005), 193-206.
6. E. Ekici, Properties of Regular set-connected Functions, Kyungpook Math. J. 44(2004), 395-403.
7. N. Ergun, On nearly paracompact spaces, Intanbul. Univ. Fen. Mec. Ser. A, 45 (1980), 65-87.

8. N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36-41.
9. N. Levine, Strong continuity in topological spaces, Amer. Math. Monthly, 67 (1960), 269.
10. P.E. Long and L.L. Herrington, Basic properties of regular closed functions, Rend. Circ. Mat. Palermo, 27 (1978), 20-28.
11. G.D. Maio, S-closed spaces, S-sets and S-continuous functions, Acad. Sci. Torino, 118 (1984), 125-134.
12. O. Njsatad, On some classes of nearly open sets, Pacific J. Math. 15 (1965), 961-970.
13. T. Noiri and G.D. Maio, Properties of α-compact spaces, Rend. Circ. Mat. Palermo. Ser II, (1998), 359-369.
14. O. Ravi, M.L. Thivagar and R. Latha, ($\alpha\hat{g}$, s)-continuous functions between topological spaces (Submitted).
15. M.K. Singal, A.R. Singal and A. Mathur, On nearly compact spaces, Boll. UMI, 4 (1969), 702-710.
16. M.H. Stone , Applications of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc. 41(1937), 375-381.
17. T. Thomson, S-closed spaces, Proc. Amer. Math. Soc. 60 (1976), 335-338.
18. M.K.R.S. Veerakumar, $\hat{g}$-closed sets in topological spaces, Bull. Allahabad Math. Soc. 18 (2003), 99-112.

****